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Quantum Effects of a Mesoscopic Capacitance Coupling Circuit With Resistances

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We study the quantization of a mesoscopic capacitance coupling circuit with resistances, derive the density matrix of the system, and study the influence of temperature on the fluctuation of the system.

KEY WORDS: dissipative capacitance coupling circuit; quantum effect; density matrix.

1. INTRODUCTION

Because of the rapid development in nanometer techniques and microelectronics, the trend in the miniaturization of integrated circuits and components toward atomic scale dimension becomes strong and definite (Buot, 1993). Clearly, when the charge-carrier inelastic coherence length and the charge-carrier confinement dimension approach the Fermi wavelength, the physics of classical devices is expected to be invalid, and quantum effects must be taken into account. The quantum effects for a single LC lossless circuit was first discussed by Louisell (1973). Following a similar line of thought, many authors have discussed the quantum effects of mesoscopic inductance coupling circuit (Song and Zhu, in press; Wang *et al.*, 2000), capacitance coupling circuit (Wang *et al.*, 2000), and two LC circuits with mutual-inductance (Fan and Pan, 1998). However, most researchers have not taken resistance of circuit into account. Because of the fact that the practical electric circuits always have resistance, the study of the quantum effects of mesoscopic circuits including resistance is very interesting. In this paper, we study the quantization of a mesoscopic capacitance coupling circuit including resistance. By introducing canonical transformation, we turn the dissipative capacitance coupling circuit into a nondissipative capacitance coupling circuit. Moreover, we derive the density matrix of the system and study the influence of temperature on the fluctuation of the system.

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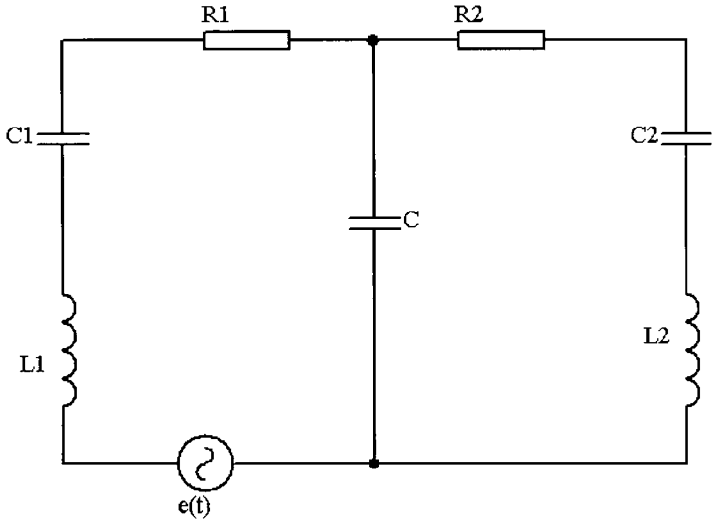

Fig. 1. Capacitance coupling circuit.

2. THE QUANTIZATION OF A MESOSCOPIC CAPACITANCE COUPLING CIRCUIT WITH RESISTANCES

We consider a mesoscopic capacitance coupling circuit with resistances (see Fig. 1). According to Kirchhoff law, the classical equations of motion of the system are

$$L_1\ddot{q}_1(t) + R_1\dot{q}_1(t) + \frac{q_1(t)}{C_1} + \frac{q_1(t) - q_2(t)}{C} = \varepsilon(t), \tag{1a}$$

$$L_2\ddot{q}_2(t) + R_2\dot{q}_2(t) + \frac{q_2(t)}{C_2} - \frac{q_1(t) - q_2(t)}{C} = 0, \tag{1b}$$

where $q_i(t)$, L_i, and C_i $(i = 1, 2)$ are the charges, inductances, and capacitances of two-component circuits, respectively, C is coupling capacitance of two-component circuit, $\varepsilon(t)$ is the electromotive force. Set $p_i(t) = L_i\dot{q}_i(t)\,(i = 1, 2)$, and $q_i(t)$, $p_i(t)$ are denoted by q_i, p_i for simplicity. So Eq. (1) can be written as

$$\dot{p}_1 = \varepsilon(t) - \frac{R_1}{L_1}p_1 - \frac{q_1}{C_1'} + \frac{q_2}{C}, \quad \dot{q}_1 = \frac{1}{L_1}p_1, \quad \frac{1}{C_1'} = \frac{1}{C_1} + \frac{1}{C}, \tag{2a}$$

$$\dot{p}_2 = -\frac{R_2}{L_2}p_2 - \frac{q_2}{C_2'} + \frac{q_1}{C}, \quad \dot{q}_2 = \frac{1}{L_2}p_2, \quad \frac{1}{C_2'} = \frac{1}{C_2} + \frac{1}{C}, \tag{2b}$$

From Eq. (2) we can obtain

$$\frac{\partial\dot{q}_1}{\partial q_1} + \frac{\partial\dot{p}_1}{\partial p_1} = -\frac{R_1}{L_1}, \quad \frac{\partial\dot{q}_2}{\partial q_2} + \frac{\partial\dot{p}_2}{\partial p_2} = -\frac{R_2}{L_2}. \tag{3}$$

Eq. (3) implies that in a dissipative system ($R_1 \neq 0$, $R_2 \neq 0$), q_i and $p_i (i = 1, 2)$ cannot be constructed as common canonical variables. When we quantize Eq. (2) in the Heisenberg picture, the equations of motion for coordinate and momentum operators share the same form as Eq. (2). But the classical variables q_i and $p_i (i = 1, 2)$ become operators, which observe appropriate commutation relations, namely the quantization condition. From the operator equation of motion corresponding to Eq. (2) we can obtain

$$\frac{dx_1}{dt} = \frac{d}{dt}(q_1 p_1 - p_1 q_1) = \dot{q}_1 p_1 + q_1 \dot{p}_1 - \dot{p}_1 q_1 - p_1 \dot{q}_1 = -\frac{R_1}{L_1} x_1 + \frac{1}{C} x_5, \quad \text{(4a)}$$

$$\frac{dx_2}{dt} = -\frac{R_2}{L_2} x_2 - \frac{1}{C} x_5, \quad \text{(4b)}$$

$$\frac{dx_3}{dt} = -\frac{R_2}{L_2} x_3 - \frac{1}{C_2'} x_5 + \frac{1}{L_1} x_6, \quad \text{(4c)}$$

$$\frac{dx_4}{dt} = -\frac{R_1}{L_1} x_4 + \frac{1}{C_1'} x_5 - \frac{1}{L_2} x_6, \quad \text{(4d)}$$

$$\frac{dx_5}{dt} = \frac{1}{L_2} x_3 - \frac{1}{L_1} x_4, \quad \text{(4e)}$$

$$\frac{dx_6}{dt} = -\frac{1}{C} x_1 + \frac{1}{C} x_2 - \frac{1}{C_1'} x_3 + \frac{1}{C_2'} x_4 - \left(\frac{R_1}{L_1} + \frac{R_2}{L_2}\right) x_6, \quad \text{(4f)}$$

where

$$x_1 = [q_1, p_1], \quad x_2 = [q_2, p_2], \quad x_3 = [q_1, p_2],$$
$$x_4 = [q_2, p_1], \quad x_5 = [q_1, q_2], \quad x_6 = [p_1, p_2]. \quad \text{(5)}$$

Generally speaking, it is very difficulty to solve Eq. (4). When $\frac{R_1}{L_1} = \frac{R_2}{L_2} = \lambda$, however, Eq. (4) has the following form of solutions:

$$x_1 = [q_1, p_1] = i\hbar \exp(-\lambda t), \quad x_2 = [q_2, p_2] = i\hbar \exp(-\lambda t) = x_1,$$
$$x_3 = [q_1, p_2] = 0, \quad x_4 = [q_2, p_1] = 0,$$
$$x_5 = [q_1, q_2] = 0, \quad x_6 = [p_1, p_2] = 0. \quad \text{(6)}$$

If we introduce the canonicalization transformation (Peng, 1980) (not the common canonical transformation) as follows:

$$Q_1(t) = q_1 \exp\left(\frac{1}{2}\lambda t\right), \quad P_1(t) = L_1 \dot{Q}_1(t) = \left(p_1 + \frac{1}{2} R_1 q_1\right) \exp\left(\frac{1}{2}\lambda t\right), \quad \text{(7a)}$$

$$Q_2(t) = q_2 \exp\left(\frac{1}{2}\lambda t\right), \quad P_2(t) = L_2 \dot{Q}_2(t) = \left(p_2 + \frac{1}{2} R_2 q_2\right) \exp\left(\frac{1}{2}\lambda t\right), \quad \text{(7b)}$$

then Eq. (1) becomes

$$L_1\ddot{Q}_1(t)+\lambda_1 Q_1(t)-\frac{1}{C}Q_2(t)=\varepsilon'(t), \tag{8a}$$

$$L_1\ddot{Q}_2(t)+\lambda_2 Q_2(t)-\frac{1}{C}Q_1(t)=0, \tag{8b}$$

or

$$\dot{Q}_1(t)=\frac{1}{L_1}P_1(t),\quad \dot{P}_1(t)=\varepsilon'(t)-\lambda_1 Q_1(t)+\frac{1}{C}Q_2(t), \tag{9a}$$

$$\dot{Q}_2(t)=\frac{1}{L_2}P_2(t),\quad \dot{P}_2(t)=-\lambda_2 Q_2(t)+\frac{1}{C}Q_1(t), \tag{9b}$$

where

$$\lambda_1=\frac{1}{C_1'}-\frac{R_1^2}{4L_1},\quad \lambda_2=\frac{1}{C_2'}-\frac{R_2^2}{4L_2},\quad \varepsilon'(t)=\varepsilon(t)\,\exp\left(\frac{1}{2}\lambda t\right). \tag{10}$$

From Eq. (9) we can prove that Q_i and P_i can be constructed as common canonical variables, here and hereafter $Q_i(t)$, $P_i(t)$ are denoted by Q_i, P_i for simplicity. By using Hamiltonian canonical equations, we obtain the classical Hamiltonian corresponding to Eq. (8)

$$H=\frac{1}{2L_1}P_1^2+\frac{1}{2L_2}p_2^2+\frac{1}{2}\lambda_1 Q_1^2+\frac{1}{2}\lambda_2 Q_2^2-\frac{1}{C}Q_1Q_2-\varepsilon'(t)Q_1, \tag{11}$$

Eq. (11) is analogous to two harmonic oscillators with a coordinate coupling term, the variables Q_1, Q_2 and P_1, P_2 play the role of coordinate and momentum of analytic mechanics.

The quantization of Eq. (11) only means that the classical variables Q_1, Q_2 and P_1, P_2 become operators. From Eqs. (6) and (7), we obtain the following commutation relations:

$$[Q_k, P_l]=i\hbar\delta_{kl},\quad [Q_1, Q_2]=[P_1, P_2]=0. \tag{12}$$

To diagonalize the Hamiltonian H of Eq. (11), we introduce the following unitary operator U:

$$U=\iint |AQ\rangle\langle Q|dQ_1dQ_2, \tag{13}$$

where

$$A=\begin{pmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}, \tag{14}$$

$$A_{11}=B\,\cos(\varphi/2),\quad A_{12}=B\,\sin(\varphi/2), \tag{15}$$

$$A_{21}=-B^{-1}\,\sin(\varphi/2),\quad A_{22}=B^{-1}\,\cos(\varphi/2), \tag{16}$$

$$B = \left(\frac{L_2}{L_1}\right)^{1/4}, \tag{17}$$

$$tg\varphi = \frac{2}{C(\lambda_1 B^2 - \lambda_2 B^{-2})}, \tag{18}$$

$$|Q\rangle = \left|\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}\right\rangle = |Q_1, Q_2\rangle = |Q_1\rangle|Q_2\rangle, \tag{19}$$

$|Q_i\rangle$ ($i = 1, 2$) is coordinate eigenstate (Fan, 1997). It is easily proved that

$$U^{-1}Q_1U = A_{11}Q_1 + A_{12}Q_2, \quad U^{-1}Q_2U = A_{21}Q_1 + A_{22}Q_2, \tag{20}$$

$$U^{-1}P_1U = A_{22}P_1 - A_{21}P_2, \quad U^{-1}P_2U = -A_{12}P_1 + A_{11}P_2. \tag{21}$$

Substituting Eqs. (20) and (21) into Eq. (11) we have

$$\begin{aligned} H' &= U^{-1}HU \\ &= \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{1}{2}m_1\omega_1^2Q_1^2 + \frac{1}{2}m_2\omega_2^2Q_2^2 - (A_{11}Q_1 + A_{12}Q_2)\varepsilon'(t), \end{aligned} \tag{22}$$

where

$$\frac{1}{m_1} = \frac{A_{22}^2}{L_1} + \frac{A_{12}^2}{L_2}, \quad \frac{1}{m_2} = \frac{A_{21}^2}{L_1} + \frac{A_{11}^2}{L_2}, \tag{23}$$

$$\omega_1^2 = \left(\frac{A_{22}^2}{L_1} + \frac{A_{12}^2}{L_2}\right)\left(\lambda_1 A_{11}^2 + \lambda_2 A_{21}^2 - \frac{2}{C}A_{11}A_{21}\right), \tag{24}$$

$$\omega_2^2 = \left(\frac{A_{21}^2}{L_1} + \frac{A_{11}^2}{L_2}\right)\left(\lambda_1 A_{12}^2 + \lambda_2 A_{22}^2 - \frac{2}{C}A_{12}A_{22}\right). \tag{25}$$

From Eq. (12) we construct the following non-Hermitian operators:

$$a_k = \sqrt{\frac{m_k\omega_k}{2\hbar}}\left(Q_k + \frac{i}{m_k\omega_k}P_k\right), \quad k = 1, 2, \tag{26}$$

$$a_k^+ = \sqrt{\frac{m_k\omega_k}{2\hbar}}\left(Q_k - \frac{i}{m_k\omega_k}P_k\right), \quad k = 1, 2, \tag{27}$$

which satisfy the following commutation relations:

$$\begin{aligned} &[a_k, a_l^+] = i\hbar\delta_{kl}, \quad [a_k, a_j] = 0, \\ &[a_k^+, a_l^+] = 0, \end{aligned} \tag{28}$$

then Eq. (22) can be rewritten as

$$H' = \hbar\omega_1\left(a_1^+a_1 + \frac{1}{2}\right) + \hbar\omega_2\left(a_2^+a_2 + \frac{1}{2}\right) + V_1(t)(a_1 + a_1^+) + V_2(t)(a_2 + a_2^+), \tag{29}$$

where

$$V_1(t) = -A_{11}\varepsilon'(t)\sqrt{\frac{\hbar}{2m_1\omega_1}}, \quad V_2(t) = -A_{12}\varepsilon'(t)\sqrt{\frac{\hbar}{2m_2\omega_2}}. \tag{30}$$

We now calculate the normal product form of U. Substituting the explicit form of the coordinate eigenstate $|Q_i\rangle$ $(i = 1, 2)$ in the Fock space

$$|Q_i\rangle = \left(\frac{m_i\omega_i}{\pi\hbar}\right)^{1/4} \exp\left[-\frac{m_i\omega_i}{2\hbar}Q_i^2 + \sqrt{\frac{2m_i\omega_i}{\hbar}}Q_i a_i^+ - \frac{1}{2}a_i^{+^2}\right]|0\rangle_i, \quad i = 1, 2 \tag{31}$$

into Eq. (13) and using the technique of the integration within an ordered product of operators (IWOP) (Fan, 1997; Fan *et al.* 1987) we can perform the integration in Eq. (13) to obtain the normal product form of U

$$\begin{aligned} U &= \iint |A_{11}Q_1 + A_{12}Q_2, A_{21}Q_1 + A_{22}Q_2\rangle\langle Q_1, Q_2| dQ_1 dQ_2 \\ &= \sqrt{\frac{4m_1m_2\omega_1\omega_2}{\hbar^2\Delta}} \exp\left(\sigma_1 a_1^{+^2} - \sigma_1 a_2^{+^2} + \sigma_2 a_1^+ a_2^+\right) \\ &\quad : \exp\left(\Gamma_1 a_1^+ a_1 + \Gamma_1 a_2^+ a_2 + \Gamma_2 a_1^+ a_2 - \Gamma_2 a_2^+ a_1\right) \\ &\quad : \exp\left(\tau_1 a_1^2 - \tau_1 a_2^2 + \tau_2 a_1 a_2\right), \end{aligned} \tag{32}$$

where : : denotes the normal ordering,

$$\begin{aligned} \Delta &= \left(\frac{m_1\omega_1 A_{12}}{\hbar}\right)^2 + \left(\frac{m_2\omega_2 A_{21}}{\hbar}\right)^2 \\ &\quad + \frac{m_1m_2\omega_1\omega_2}{\hbar^2}[2 + (B^2 + B^{-2})\cos^2(\varphi/2)], \end{aligned} \tag{33}$$

$$\sigma_1 = \frac{m_1\omega_1}{\Delta\hbar^2}\left[m_1\omega_1 A_{12}^2 + m_2\omega_2\left(1 + A_{11}^2\right)\right] - \frac{1}{2}, \tag{34}$$

$$\sigma_2 = \frac{\sqrt{m_1m_2\omega_1\omega_2}}{\Delta\hbar^2}(m_1\omega_1 - m_2\omega_2)\sin(\varphi), \tag{35}$$

$$\Gamma_1 = \frac{2m_1m_2\omega_1\omega_2}{\Delta\hbar^2}(B + B^{-1})\cos(\varphi/2) - 1, \tag{36}$$

$$\Gamma_2 = \frac{\sqrt{4m_1m_2\omega_1\omega_2}}{\Delta\hbar^2}(m_1\omega_1 B + m_2\omega_2 B^{-1})\sin(\varphi/2), \tag{37}$$

$$\tau_1 = \frac{m_1\omega_1}{\Delta\hbar^2}\left[m_1\omega_1 A_{12}^2 + m_2\omega_2\left(1 + A_{22}^2\right)\right] - \frac{1}{2}, \tag{38}$$

$$\tau_2 = -\frac{\sqrt{m_1m_2\omega_1\omega_2}}{\Delta\hbar^2}(m_1\omega_1 B^2 - m_2\omega_2 B^{-2})\sin(\varphi). \tag{39}$$

It can be proved that the time evolution operator $U_s(t, 0)$ corresponding to H' is given by (Fan, 1997)

$$U_s(t, 0) = \exp\lfloor -i\hbar(\omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2)t \rfloor \\ \exp\lfloor -i(\eta_1^* a_1^+ + \eta_1 a_1 + \eta_2^* a_2^* + \eta_2 a_2)\rfloor, \quad (40)$$

here we have omitted a phase factor,

$$\eta_k(t) = \int_0^t V_k(\tau)\exp(-i\hbar\omega_k\tau)d\tau. \quad (41)$$

Therefore, the wave function of the system at time t is given by

$$|\psi(t)\rangle = UU_s(t, 0)|00\rangle, \quad (42)$$

here we suppose that the initial state of the system is in two-mode vacuum state $|00\rangle$. When the external electric source is only instantaneously switched on, say, for an infinitesimal time $t = \rho \to 0$ and then switched off, the system is in a rotated two single-mode squeezed state

$$\begin{aligned} |\psi(t = \rho)\rangle_{p\to 0} &= U|00\rangle \\ &= \sqrt{\frac{4m_1m_2\omega_1\omega_2}{\hbar^2\Delta}}\exp\left(\sigma_1 a_1^{+^2} - \sigma_1 a_2^{+^2} + \sigma_2 a_1^+ a_2^+\right)|00\rangle \\ &= \sqrt{\frac{4m_1m_2\omega_1\omega_2}{\hbar^2\Delta}}\exp\left[(a_1^+ a_2 - a_2^+ a_1)\theta\right]\exp\left[\nu\left(a_2^{+^2} - a_1^{+^2}\right)\right]|00\rangle, \end{aligned} \quad (43)$$

where

$$\theta = \frac{1}{2}\arctan\left(-\frac{\sigma_2}{2\sigma_1}\right), \quad \nu = \sigma_1 + \frac{1}{2}\sigma_2\cot(\theta), \quad (44)$$

$\exp\lfloor(a_1^+ a_2 - a_2^+ a_1)\theta\rfloor$ is a rotation operator. From Eqs. (20) and (21) we have

$$\begin{aligned} \langle(\Delta Q_1)^2\rangle &= \left\langle 00\middle| U^+ Q_1^2 U \middle| 00\right\rangle - (\langle 00|U^+ Q_1 U|00\rangle)^2 \\ &= \frac{\hbar A_{11}^2}{2m_1\omega_1} + \frac{\hbar A_{12}^2}{2m_2\omega_2}, \end{aligned} \quad (45)$$

$$\begin{aligned} \langle(\Delta Q_2)^2\rangle &= \left\langle 00\middle| U^+ Q_2^2 U \middle| 00\right\rangle - (\langle 00|U^+ Q_2 U|00\rangle)^2 \\ &= \frac{\hbar A_{21}^2}{2m_1\omega_1} + \frac{\hbar A_{22}^2}{2m_2\omega_2}, \end{aligned} \quad (46)$$

$$\begin{aligned} \langle(\Delta P_1)^2\rangle &= \left\langle 00\middle| U^+ P_1^2 U \middle| 00\right\rangle - (\langle 00|U^+ P_1 U|00\rangle)^2 \\ &= \frac{\hbar m_1\omega_1 A_{22}^2}{2} + \frac{\hbar m_2\omega_2 A_{21}^2}{2}, \end{aligned} \quad (47)$$

$$\langle(\Delta P_2)^2\rangle = \left\langle 00\middle|U^+ P_2^2 U\middle|00\right\rangle - (\langle 00|U^+ P_2 U|00\rangle)^2$$
$$= \frac{\hbar m_1 \omega_1 A_{12}^2}{2} + \frac{\hbar m_2 \omega_2 A_{11}^2}{2}, \tag{48}$$

From Eq. (4) we obtain the quantum fluctuations of q_i and p_i $(i = 1, 2)$ in the state $U|00\rangle$ as follows:

$$\begin{aligned}\langle(\Delta q_1)^2\rangle &= \left\langle q_1^2\right\rangle - \langle q_1\rangle^2\\ &= \left[\left\langle 00\middle|U^+ Q_1^2 U\middle|00\right\rangle - (\langle 00|U^+ Q_1 U|00\rangle)^2\right] \exp(-\lambda t)|_{t=\rho\to 0}\\ &= \langle(\Delta Q_1)^2\rangle,\end{aligned} \tag{49}$$

$$\langle(\Delta q_2)^2\rangle = \langle(\Delta Q_2)^2\rangle, \tag{50}$$

$$\langle(\Delta p_1)^2\rangle = \langle(\Delta P_1)^2\rangle, \tag{51}$$

$$\langle(\Delta p_2)^2\rangle = \langle(\Delta P_2)^2\rangle, \tag{52}$$

3. DENSITY MATRIX AND ENSEMBLE AVERAGE OF THE SYSTEM

To study the influence of temperature on the fluctuation of the system, we now calculate the density matrix of the system. Usually, in the coordinate representation, a density matrix $\rho(x, x'; \beta)$ is calculated by the Bloch equation

$$-\frac{\partial}{\partial\beta}\rho(x, x'; \beta) = H\rho(x, x'; \beta), \tag{53}$$

where $\beta = (k_B T)^{-1}$, k_B is the Boltzmann constant. Here, we use the unitary transformation U to calculate the density matrix $\rho(Q_1, Q_2; Q_1', Q_2'; \beta)$ of the system. For simplicity, we only consider the case $\varepsilon(t) = 0$. From Eqs. (20)–(22) we have

$$\begin{aligned}\rho(Q_1, Q_2; Q_1', Q_2'; \beta) &= \langle Q_1, Q_2|\exp(-\beta H)|Q_1', Q_2'\rangle\\ &= \langle Q_1, Q_2|U \exp(-\beta H')U^+|Q_1', Q_2'\rangle\\ &= \langle A_{22}Q_1 - A_{12}Q_2, -A_{21}Q_1 + A_{11}Q_2|\exp(-\beta H')|\\ &\quad \times A_{22}Q_1' - A_{12}Q_2', -A_{21}Q_1' + A_{11}Q_2'\rangle,\end{aligned} \tag{54}$$

where we have used $U^+|Q_1, Q_2\rangle = |A^{-1}\binom{Q_1}{Q_2}\rangle$. Because the position representation of the density matrix of a single harmonic oscillator is known as (Mills and Robiette, 1985)

$$\langle x|\exp\left[-\beta\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2\right)\right]|x'\rangle = \left(\frac{m\omega}{2\pi\hbar\sinh(\hbar\omega\beta)}\right)^{1/2}$$
$$\exp\left[-\frac{m\omega}{2\hbar\sinh(\hbar\omega\beta)}((x^2 + x'^2)\cosh(\hbar\omega\beta) - 2xx')\right], \tag{55}$$

we have

$$\rho(Q_1, Q_2; Q_1', Q_2'; \beta) = \langle Q_1, Q_2| \exp(-\beta H)| Q_1', Q_2'\rangle$$

$$= \frac{1}{2\pi\hbar}\left(\frac{m_1 m_2 \omega_1 \omega_2}{\sinh(\hbar\omega_1\beta)\sinh(\hbar\omega_2\beta)}\right)^{1/2} \exp\left\{-\frac{m_1\omega_1}{2\hbar\sinh(\hbar\omega_1\beta)}\right.$$

$$\times [((A_{22}Q_1 - A_{12}Q_2)^2 + (A_{22}Q_1' - A_{12}Q_2')^2)\cosh(\hbar\omega_1\beta)$$

$$- 2(A_{22}Q_1 - A_{12}Q_2)(A_{22}Q_1' - A_{12}Q_2')]$$

$$- \frac{m_2\omega_2}{2\hbar\sinh(\hbar\omega_2\beta)}[((-A_{21}Q_1 + A_{11}Q_2)^2$$

$$+ (-A_{21}Q_1' + A_{11}Q_2')^2)\cosh(\hbar\omega_2\beta)$$

$$\left. - 2(-A_{21}Q_1 + A_{11}Q_2)(-A_{21}Q_1' + A_{11}Q_2')] - \right\}. \quad (56)$$

Similarly, we can calculate $\rho(P_1, P_2; P_1', P_2'; \beta)$ in the momentum representation

$$\rho(P_1, P_2; P_1', P_2'; \beta) = \langle P_1, P_2| \exp(-\beta H)| P_1', P_2'\rangle$$

$$= \langle p_1, p_2| U \exp(-\beta H')U^+ | p_1', p_2'\rangle$$

$$= \langle A_{11}P_1 + A_{21}P_2, A_{12}P_1 + A_{22}P_2| \exp(-\beta H')| A_{11}P_1'$$

$$+ A_{21}P_2', A_{12}P_1' + A_{22}P_2'\rangle. \quad (57)$$

By using the momentum representation of the density matrix of a single harmonic oscillator (Mills and Robiette, 1985)

$$\langle p| \exp\left[-\beta\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2\right)\right]| p'\rangle = \left(\frac{1}{2\pi\hbar m\omega\sinh(\hbar\omega\beta)}\right)^{1/2}$$

$$\exp\left[-\frac{1}{2\hbar m\omega\sinh(\hbar\omega\beta)}((p^2 + p'^2)\cosh(\hbar\omega\beta) - 2pp')\right], \quad (58)$$

we have

$$\rho(P_1, P_2; P_1', P_2'; \beta) = \langle P_1, P_2| \exp(-\beta H)| P_1', P_2'\rangle$$

$$= \frac{1}{2\pi\hbar}\left(\frac{1}{m_1 m_2 \omega_1 \omega_2 \sinh(\hbar\omega_1\beta)\ \sinh(\hbar\omega_2\beta)}\right)^{1/2}$$

$$\times \exp\left\{-\frac{1}{2\hbar m_1\omega_1\sinh(\hbar\omega_1\beta)}[((A_{11}P_1 + A_{21}P_2)^2\right.$$

$$+ (A_{11}P_1' + A_{21}P_2')^2)\cosh(\hbar\omega_1\beta) - 2(A_{11}P_1 + A_{21}P_2)$$

$$\times (A_{11}P_1' + A_{21}P_2')] - \frac{1}{2\hbar m_2\omega_2\ \sinh(\hbar\omega_2\beta)}$$

$$\times [((A_{12}P_1 + A_{22}P_2)^2 + (A_{12}P_1' + A_{22}P_2')^2)\cosh(\hbar\omega_2\beta) - 2(A_{12}P_1 + A_{22}P_2)(A_{12}P_1' + A_{22}P_2')] - \Big\}. \quad (59)$$

From Eqs. (56) and (59) we have

$$\langle(\Delta Q_1)^2\rangle = \left\langle Q_1^2\right\rangle - \langle Q_1\rangle^2 = \frac{\int\int Q_1^2\rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}{\int\int \rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2} - \left(\frac{\int\int Q_1\rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}{\int\int \rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}\right)^2 = \frac{\hbar A_{11}^2}{2m_1\omega_1\tanh\left(\frac{\hbar\omega_1\beta}{2}\right)} + \frac{\hbar A_{12}^2}{2m_2\omega_2\tanh\left(\frac{\hbar\omega_2\beta}{2}\right)}, \quad (60)$$

$$\langle(\Delta Q_2)^2\rangle = \left\langle Q_2^2\right\rangle - \langle Q_2\rangle^2 = \frac{\int\int Q_2^2\rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}{\int\int \rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2} - \left(\frac{\int\int Q_2\rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}{\int\int \rho(Q_1, Q_2; Q_1, Q_2;\beta)dQ_1dQ_2}\right)^2 = \frac{\hbar A_{21}^2}{2m_1\omega_1\tanh\left(\frac{\hbar\omega_1\beta}{2}\right)} + \frac{\hbar A_{22}^2}{2m_2\omega_2\tanh\left(\frac{\hbar\omega_2\beta}{2}\right)}, \quad (61)$$

$$\langle(\Delta P_1)^2\rangle = \left\langle P_1^2\right\rangle - \langle P_1\rangle^2 = \frac{\int\int P_1^2\rho(P_1, P_2; P_1, P_2;\beta)dP_1dP_2}{\int\int \rho(P_1, P_2; P_1, P_2;\beta)dP_1dP_2}$$

$$- \left(\frac{\iint P_1 \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2}{\iint \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2} \right)^2$$

$$= \frac{\hbar m_1 \omega_1 A_{22}^2}{2 \tanh\left(\frac{\hbar \omega_1 \beta}{2}\right)} + \frac{\hbar m_2 \omega_2 A_{21}^2}{2 \tanh\left(\frac{\hbar \omega_2 \beta}{2}\right)}, \tag{62}$$

$$\langle (\Delta P_2)^2 \rangle = \left\langle P_2^2 \right\rangle - \langle P_2 \rangle^2$$

$$= \frac{\iint P_2^2 \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2}{\iint \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2}$$

$$- \left(\frac{\iint P_2 \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2}{\iint \rho(P_1, P_2; P_1, P_2; \beta) dP_1 dP_2} \right)^2$$

$$= \frac{\hbar m_1 \omega_1 A_{12}^2}{2 \tanh\left(\frac{\hbar \omega_1 \beta}{2}\right)} + \frac{\hbar m_2 \omega_2 A_{11}^2}{2 \tanh\left(\frac{\hbar \omega_2 \beta}{2}\right)} \tag{63}$$

From Eq. (7) we obtain the fluctuations of charges q_1 and q_2 as follows:

$$\langle (\Delta q_1)^2 \rangle = \left\langle q_1^2 \right\rangle - \langle q_1 \rangle^2$$

$$= \left[\frac{\hbar A_{11}^2}{2 m_1 \omega_1 \tanh\left(\frac{\hbar \omega_1 \beta}{2}\right)} + \frac{\hbar A_{12}^2}{2 m_2 \omega_2 \tanh\left(\frac{\hbar \omega_2 \beta}{2}\right)} \right] \exp(-\lambda t), \tag{64}$$

$$\langle (\Delta q_2)^2 \rangle = \left\langle q_2^2 \right\rangle - \langle q_2 \rangle^2$$

$$= \left[\frac{\hbar A_{21}^2}{2 m_1 \omega_1 \tanh\left(\frac{\hbar \omega_1 \beta}{2}\right)} + \frac{\hbar A_{22}^2}{2 m_2 \omega_2 \tanh\left(\frac{\hbar \omega_2 \beta}{2}\right)} \right] \exp(-\lambda t), \tag{65}$$

From the the property of the function $\tanh(x)$ we can see that the fluctuations of the charges q_1 and q_2 increase with increasing temperature T. As $T \to 0$ and $t \to 0$, Eqs. (64) and (65) turn back to Eqs. (49) and (50).

4. CONCLUSION

By introducing canonicalization transformation (not the common canonical transformation), we study the quantization of a mesoscopic capacitance coupling circuit with resistances. By taking a unitary transformation approach, we turn the system into two independent harmonic oscillators, then we discuss the time evolution of the system. It is remarkable that when the external electric source is only instantaneously switched on, say, for an infinitesimal time $t = \rho \to 0$ and then switched off, the system is in a rotated squeezed vacuum state. To study the influence of temperature on the fluctuation of the system, we calculate the density matrix of the system. The results show that the fluctuations of the charges q_1 and q_2 increase with increasing temperature T.

It should be pointed out that our discussion is only confined to the special case of $\frac{R_1}{L_1} = \frac{R_2}{L_2} = \lambda$. The study of the system in general case would be very interesting. But we have trouble in solving Eq. (4).

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